Diagnostic Analysis of Operations in a Manufacturing System Using Goal Programming Model

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Abstract: *Many manufacturing industries in this part of the globe are faced with lack of reliable Decision Support Systems (DSS) for operations planning and control. Having identified the job, operation or system beckoning for improvement study, the next steps involve is the development, solving and implementation of the recommendations from the result of solved model(s). Having analyzed different aspects of applications of mathematical modelling in operations managements, this article illustrated the development and application of goal programming model in diagnostic analysis of operations in manufacturing system, setting production targets in a selected manufacturing industry with default production characteristics. Some real life interpretation of some of the models' parameters and components in the application area were discussed with the view of enhancing efficiency of production planning and control.*

Keywords: *Operations Managements, Mathematical Modelling, Goal Programming, Decision Support System.*

1. Introduction

Operations management is the activity of managing the resources used in production of goods and delivering of services [1]. It is basically a well-articulated decision making task of choosing what to do, when and how to do them, from the feasible solution sets. The efficiency of planning and scheduling in manufacturing, as well as distribution systems is considered vital towards the success of production-based businesses. This is seen in tackling practical problem of setting effective product mix production targets.

The complexity and dynamic nature of manufacturing systems demand robust and sophisticated methods of diagnostic analysis to ensure optimal performance, reliability, and productivity. Manufacturing systems often need to balance multiple, sometimes conflicting objectives such as cost reduction, quality improvement, timely delivery, and resource utilization. Goal programming (GP), a branch of multi-objective optimization, offers a structured approach to address these challenges by allowing the simultaneous consideration of multiple goals and minimizing deviations from these goals.

The primary types of GP, in terms of underlying distance metric, include Weighted Goal Programming (WGP), Lexicographic Goal Programming (LGP), and Chebyshev Goal Programming (CGP), which provide different frameworks for prioritizing and balancing these objectives [2]. This flexibility makes GP particularly suitable for complex manufacturing environments where trade-offs between conflicting objectives are common. In production planning and scheduling, goal programming has proven to be effective in optimizing production processes while balancing multiple objectives such as cost, time, and resource utilization.

For instance**,** Ayomoh [3] employed GP in determination of optimal model with multiple activities of different priority orders. Similarly, Jayaraman, et al. [4] employed a Weighted Goal Programming model involving criteria on the economic development (GDP), the electricity consumption, the greenhouse gas emissions, and the total number of employees in an attempt to determine optimal labour allocation across various economic sectors. Recently, Gütmen, Roy and Weber [5] presented a comparative analysis of GP and WGP application in multi-objective transportation problem. Such application was successfully carried out in quality control of waste treatment systems [6], formulation for the optimal planning of the daily production of sawmills [7], Contract Pricing of Electric Vehicle Aggregator in Join Day-Ahead Market [8], resolution of Production Planning Problems in textile production system [9], and beyond. More so, combined goal programming and inverse data envelopment analysis (DEA) method for target setting in mergers was presented and applied to banking industries [10]. This illustrates versatility and effectiveness of GP in tackling planning challenges in various sectors.

The importance of effective planning as a panacea for operational efficiency has been highlighted in [11 and 12]. This paper then presents a class of product mix optimization problems in production planning, and adopted goal programming approach in seeking satisfactory solution based on the developed mathematical model.

2. The Product Mix Optimization Problem

In this paper, polyvinyl chloride pipe manufacturing system with a very low productivity index which affects the company economically was technically analyzed from economic standpoint. The product mix optimization problem for setting monthly production target in a manufacturing system was considered. Below are the needed attributes of the system which were obtained from the work study record of the system:

(1) Set of products to be produced,
(2) the upper or/and the lower limits of production quantity for each product,

(3) Sequence of facilities/processes required for completing each product
(4) the processing time on each facility
(5) Capacity and availability of each facility
(6) Selling price and material cost for a unit of each product
(7) Cost for a unit of utilization of each facility

The problem is to determine the product mix, i.e., the quantity of each product to be produced, such that the marginal profit will be maximized, while satisfying the constraints on the demand for each product and the availability of each facility.

Goal programming in its simplest form involves setting of goals for each objective that is to be attained. The optimum solution **X*** is then defined as the one that minimizes the deviations from the set goals. Thus the goal programming formulation of the multi-objective optimization problem leads to,

Minimize $\left[\sum_{j=1}^{k}(d_j^+ + d_j^-)^p\right]^{1/p}$, $p \geq 1$

Subject to:

$$f_j(\mathbf{X}) + d_j^+ + d_j^- = b_j \quad j = 1, 2, \ldots, k$$

$$d_j^+, d_j^- \geq 0 \quad \text{for } j = 1, 2, \ldots, k$$

$$d_j^+ * d_j^- = 0 \quad \text{for } j = 1, 2, \ldots, k$$

Where b_j is the goal set by the designer for the jth objective and d_j^+ and d_j^- are the underachievement and overachievement of the jth goal respectively. The value of p is based on the utility function chosen by the designer. Often the goal for the jth objective, b_j, is found by first solving the following problem:

Minimize $f_j(X)$

Subject to

$$g_j(X) \leq 0, \quad j = 1, 2, \ldots, m$$

If the solution of the problem stated above is denoted by X_j^*, then bj is taken as:

$$b_j = f_j(X_j^*)$$

3. Model Development

One of the goal equations in the goal programming model was developed by running the break even analysis on the system. The model when solved will give the monthly target volume of each product, which in turn be used to set production targets for the workers. Thus we developed the breakeven point equation from the first principle,

Total Revenue (TR) = Total Cost (TC) at breakeven point. (1)

Under the following Assumption we thus develop equation for finding the breakeven volume for the multi-product system

$g_j(\mathbf{X}) \leq 0$, j

1. The only source of revenue is from sales,
2. The cost components could be classified as fixed and variable. Where the fixed cost consist of rent, depreciation charges on machines, workers‟ wages, fuel cost, and electricity bill, this does not depend on activity level e.g. production volume. The material cost, operation cost that depends on the activity level has been classified as variable

cost as observed for the system under study.
3. These costs are deterministic entities
4. Total volume produced were sold, as obtainable in make to order production

$$\sum_{i=1}^{m} usp_i\, v_i = Fc + \sum_{i=1}^{m} uvc_i\, v_i \qquad (2)$$

Where,

usp_i = unit selling price of product i

uvc_i = unit variable cost of product i

v_i = volume of product i produced or sold

m = number of various product considered.

Now, considering the system with stipulated acceptable level of Return on Investment (ROI)an accounting-based approach, just like internal rate of return (IRR or ROR)which is usually the measured against minimum acceptable rate of return (MARR) in the cash flow to ascertain the viability of a project. From the definition of the ROI as the ratio of the average annual accounting profit to the original book value of the asset. It could be applied to the system under consideration as the ratio of the average periodical (say monthly, annually etc.) accounting profit (i.e. Total Revenue (TR) – Total Cost (TC)) to the total cost of production.

Thus we have,

$$\frac{TR - TC}{TC} = ROI \qquad (3)$$

Let α be the recommended value for overall ROI in fraction. Substituting this in equation 3 and making TR subject of formular, we have,

$$TR = (1+\alpha)TC \equiv (1+\alpha)(FC + \sum_{i=1}^{m} uvc_i\, v_i) \qquad (4)$$

This implies that,

$$\sum_{i=1}^{m} usp_i\, v_i - (1+\alpha)\sum_{i=1}^{m} uvc_i\, v_i = (1+\alpha)FC \qquad (5)$$

The other goal is the improvement of machine utilization time to β % of the total available machine time. This implies that only (1- β) % of the available time is permitted as machine idle time (including set up time) during production period. Where T is the total available machine processing time (in hours) and t_i represents the unit machine processing time (in hours) for product i.

$$\sum_{i=1}^{m} t_i\, v_i = \beta T \qquad (6a)$$

And,

$$T = k * l * n \qquad (6b)$$

Where, k is the number of working days per study period, l is the available working hours per day and n is the number of available machine simultaneously processing the product.

This gives rise to equation 7,

$$\sum_{i=1}^{m} t_i\, v_i = \beta kln \qquad (7)$$

The two constraints developed in our model are restriction placed by raw material availability and the product demand. In the case in hand, we have *'make to order'* system and thus must not exceed demand in accordance to the 4th assumption.

$$\sum_{i=1}^{m} r_{ij}\, v_i \le R_j \qquad \forall j \qquad (8)$$

$$v_i \le Q_i \qquad \forall i \qquad (9)$$

Where, r_{ij} is quantity of raw material j used in producing product i, R_j is the total available quantity of raw material j and Q_i is the demand of product i for the period under study. Expansion of inequalities 8 and 9 will yield a total of (i + j) inequalities as constraints of the goal programming model, while equation 5 and 7 forms the goals. The following data in Table 1a and Table 1b were obtained from the PVC pipe producing company under study.

The model below was obtained by expounding and feeding in the values of the parameter, in the general model,

$$v_1 + 2v_2 - 80v_3 - 44v_4 = 585000$$

$$0.0186v_1 + 0.0417v_2 + 0.0833v_3 + 0.0333v_4 = 1020$$

$$0.195v_1 + 1.214v_2 + 5.26v_3 + 1.305v_4 \le 2500$$

$$0.004v_1 + 0.004v_2 + 0.175v_3 + 0.026v_4 \le 50$$

$$0.005v_1 + 0.234v_2 + 1.052v_3 + 0.261v_4 \le 500$$

$$0.0026v_1 + 0.0016v_2 + 0.07v_3 + 0.017v_4 \le 50$$

$$0.000051v_1 + 0.0011v_2 + 0.005v_3 + 0.001v_4 \le 100$$

$$0.052v_1 + 0.005v_2 + 0.021v_3 + 0.005v_4 \le 100$$

$$v_1 \le 1190$$

$$v_2 \le 300$$

$$v_3 \le 515$$

$$v_4 \le 600$$

$$v_1, v_2, v_3, v_4 \ge 0$$

Table 1a: The values of the model parameters obtained from work study

Product	usp_i (₦)	uvc_i (₦)	Q_i	t_i(hrs)	r_{11}	r_{12}	r_{13}	r_{14}	r_{15}	r_{16}
1	105	80	1190	0.0186	0.195	0.004	0.005	0.00026	0.000051	0.052
2	470	360	300	0.0417	1.214	0.004	0.234	0.0016	0.0011	0.005
3	2000	1600	515	0.0833	5.26	0.175	1.052	0.07	0.005	0.021
4	450	380	600	0.0333	1.305	0.026	0.261	0.017	0.001	0.005

Table 1b: The values of the model parameters obtained from work study

FC (₦)	β(%)	k	l(hrs)	n(units)	A	M	R_1(kg)	R_2 (kg)	R_3 (kg)	R_4 (kg)	R_5 (kg)	R_6 (kg)
450,000	85	25	18	3	0.3	5	2500	50	500	50	100	100

Table 2: The Summary of Result Obtained from the QM Solution software

Decision variable analysis	Value	Priority analysis	Non achievement
v1	1190	Priority 1	583210
v2	300		
v3	0	Priority 2	985.4
v4	0		
Constraint Analysis	RHS	d+ (rowi)	d- (rowi)
Goal/ Cnstrnt 1	585000	0	583210
Goal/ Cnstrnt 2	1020	0	985. 4
Goal/ Cnstrnt 3	2500	0	1903. 8
Goal/ Cnstrnt 4	50	0	44. 0
Goal/ Cnstrnt 5	500	0	423. 9
Goal/ Cnstrnt 6	50	0	49. 2
Goal/ Cnstrnt 7	100	0	99.6
Goal/ Cnstrnt 8	100	0	36.6
Goal/ Cnstrnt 9	1190	0	0
Goal/ Cnstrnt 10	300	0	0
Goal/ Cnstrnt 11	515	0	515
Goal/ Cnstrnt 12	600	0	600

3. Results and Discussion

From the above result in Table 2, obtained using QM software, the following deduction could be drawn as pertaining to the challenges facing the system,

Decision variable, Priority and Constraint Analysis

The result suggests production of only product 1 and 2 to the tune of the total monthly quantity of 1190 and 300 units respectively. Products 3 and 4 should not be produced, considering negative contribution they pose to the actualization of the desired return on investment, as shown in first line of the model which was derived from equation 5. This could be remedied by improving the value of

$$usp_i v_i - (1+\alpha) uvc_i v_i \qquad i = 3,4$$

The improvement could be achieved by reducing uvci or/and α or/and increasing uspi. But from the productivity point of view, the option of increasing uspi would not be a very good solution, and the company policy on the acceptable return on investment may not be an easy nut to crack with respect to reduction of α. Thus the best feasible solution will be to reduce the value of uvci. This could be done by buying in bulk, reduction on material waste, or redesign of the production process with respect to the material consumption. The above expression must have positive value for product 3 and 4 to have a positive contribution to the return on investment.

From the result shown above in Table 2, it could be seen that the two goals of the model were underachieved. It was only about 3.39% of the available production time that was utilized, and only about 0.31% of the RHS of goal 1(return on the investment) was achieved. This explains the high level of machine redundancy observed during the work-study, and justifies the claims of the management of very poor return on investment which to very great extent resulted in poor productivity of the system. From the value obtained from the goal programming result, it could be deduced that one out the three machines could handle production within the one month study period and yet be under-utilized to the tune of 89.82% of the total available machine processing time. Only two of the four products were to be produced, thus giving rise to the poor machine utilization. More so production of product 3 and 4 may increase machine utilization index but negatively affect the return on investment. Hence there is need to bring product 3 and 4 into the optimal solution mix.

The result in Table 2 also shows that all the constraints' RHS were under achieved except constraints 9 and 10. Constraints 3 to 8, reveals that on the average, 80.6% of the monthly available raw material were unutilized. The table also shows that the monthly demand on product 1 and 2 were achieved, but that of product 3 and 4 were not achieved at all.

The diagnosis shows that only product 1 and 2 to the tune of the total monthly quantity of 1190 and 300 units respectively are economically reasonable to produce. The negative contribution Products 3 and 4 pose to the actualization of the desired return on investment has shown that market expansion or increase in the demand of these products is not enough to solve the problem of the system .

The poor return on investment which to very great extent resulted in poor productivity of the system not minding all effort by the company to increase production was diagnosed to be caused by the negative contribution of Products 3 and 4 whose production quantity was also improved alongside with the other products. Therefore, expanding the market for product 1 and 2 and eliminating product 3 and 4 from the product mix will simultaneously solve both the problem of machine utilization and return on investment. This will be another means of optimizing the system if product 3 and 4 are not among those classes of goods that must be produced for goodwill sake and does not have complementary demand with product 1 and 2 which will affect customers' decision on patronage.

4. Conclusion

The key ailing areas of the manufacturing system were identified using the application of system analysis techniques and modelling. This was achieved by the development of a reliable Decision Support System (DSS) that is borne out of systematic analysis and synthesis of the system's problem and the use of goal programming (where the expected output or returns were used in developing the goal, and the prevailing characteristics of the system served as set of constraints to be solved). The result of the diagnosis show that only product 1 and 2 to the tune of the total monthly quantity of 1190 and 300 units respectively are economically viable, considering the negative contribution products 3 and 4 pose to the actualization of the desired return on investment, as shown in first line of the model which was derived from equation 5. It was also seen that only about 3.39% of the available production time will be utilized, and only about 0.31% of the RHS of the goal 1(return on the investment) will be achieved when these quantity are produced. The claim of high level of machine redundancy and the poor return on investment which to very great extent resulted in poor productivity of the system were confirmed. Only two of the four products were to be produced, thus giving rise to the poor machine utilization. More so production of product 3 and 4 may increase machine utilization index but negatively affect the return on investment. There was a need to bring product 3 and 4 into the optimal solution mix by reducing the value of uvc_i or eliminate product 3 and 4 from the mix and have the demand for product 1 and 2 increased in order to improve the return on the investment as well as the machine utilization index. After critical analysis it was found that through application of some business process reengineering techniques, problems of ailing production system could be diagnosed and some productivity solutions suggested.

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